E 675 (Pages : 3) Reg. No.....

Name.....

B.Sc. DEGREE (CBCSS) EXAMINATION, NOVEMBER 2010

Third Semester

Core Course—CALCULUS

(Common for Model I and Model II B.Sc. Mathematics and B.Sc. Computer Applications (Three Main))

Time : Three Hours Maximum Weight : 25

Part A (Objective Type Questions)

Answer **all** *questions.*

Each bunch of four questions has weight 1.

I. 1 If $y = \frac{1}{ax+b}$, then the nth derivative $y_n =$ ———.

2 The $(n+1)$th term in the expansion of $f(a+h)$ is ———.

3 The curve $y = f(x)$ is ——— if $\frac{d^2y}{dx^2} > 0$.

4 The locus of centre of curvature is known as ———.

II. 5 Find the asymptotes of the curve $x^2y - y - x = 0$ parallel to the y-axis.

6 Find $\frac{\partial f}{\partial y}$ if $f(x, y) = \sin(xy)$.

7 If $w = f(x, y, z)$ is differentiable and x, y and z are differentiable functions of t, then $\frac{dw}{dt} =$ ———.

8 Find the critical point of $f(x, y) = xy$.

III. 9 Evaluate $\int_0^{\pi/4} \tan x \sec^2 x \, dx$.

10 State Shell formula for the volume of the solid generated about a vertical line.

11 Write the formula for the length of $y = f(x)$, $a \le x \le b$.

12 Write the formula for the surface area generated by revolving the curve $x = f(t)$, $y = g(t)$, $a \le t \le b$, about the x-axis.

IV. 13 Define area of a closed bounded region R using a double integral.

14 Change the Cartesian integral $\iint_R f(x,y)\, dx\, dy$ into a polar integral.

Turn over

15 What is the average value of a function F over a region D in space ?

16 Write the equation in cylindrical co-ordinates of a cylinder of radius 5 units with z-axis as its axis.

$(4 \times 1 = 4)$

Part B (Short Answer Type Questions)

Answer any **five** *questions.*
Each question has weight 1.

17 Expand e^x as a Maclaurin's series.

18 Find the envelope of the family of lines $y = mx + \sqrt{1+m^2}$, m being the parameter.

19 Find $\frac{dw}{dt}$ at $t = 0$, if $w = x^2y - y^2$, $x = \sin t$ and $z = t$.

20 Define Saddle point of a surface $z = f(x, y)$.

21 Find the area of the region enclosed by the curve $x = y^2$ and the line $x = y + 2$, integrating with respect to y.

22 Find the volume of the solid generated by revolving the region under the curve $y = \sqrt{x}$, between the lines $x = 1$ and $x = 4$, about the x-axis.

23 Evaluate $\iint_R \frac{\sin x}{x}\, dA$, where R is the region in the xy-plane bounded by the x-axis, the line $y = x$ and the line $x = 1$.

24 Evaluate $\int_0^1 \int_0^{1-y} \int_0^2 dx\, dz\, dy$.

$(5 \times 1 = 5)$

Part C (Short Essay Questions)

Answer any **four** *questions.*
Each question has weight 2.

25 Find the centre of curvature at any point $\left(a\cos^3\theta, a\sin^3\theta\right)$ on the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

26 Find the asymptotes of the curve $x^3 + y^3 = 3axy$.

27 Find the local extreme values of the function $f(x, y) = x^2 + xy + y^2 + 3x - 3y + 4 = 0$.

28 The region bounded by the curve $y = \sqrt{4x - x^2}$, the x-axis, and the line $x = 2$ is revolved about the x-axis to generate a solid. Find the volume of the solid.

29 Applying surface area formula find the area of the surface generated by revolving the circle $x^2 + y^2 = a^2$, about the x-axis.

30 Use polar co-ordinates to evaluate $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} \left(x^2 + y^2\right) dy\, dx$.

$(4 \times 2 = 8)$

Part D (Essay Questions)

Answer any **two** *questions.*
Each question has weight 4.

31 (a) State Leibnitz's theorem (without proof) on the nth derivative of a product of two functions.

(b) If $y = \cos(m \sin^{-1} x)$, show that $\left(1-x^2\right)y_{n+2} - (2n+1)xy_{n+1} + \left(m^2-n^2\right)y_n = 0$ and hence find $y_n(0)$.

32 Find the length of the curve $y = \left(\frac{x}{2}\right)^{2/3}$ from $x = 0$ to $x = 2$.

33 (a) Find a spherical co-ordinate equation for the sphere $x^2 + (y-1)^2 + z^2 = 1$.

(b) Evaluate $\int_0^{2\pi} \int_0^{\pi/3} \int_{\sec\phi}^{2} 3\rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$.

$(2 \times 4 = 8)$